Vidyasagar College for Women

Internal Examination 2020

Mathematics (Hons.) MTMA Part-I

Full Marks: 100 Module-III and IV Time: 2 Hrs

Use separate answer scripts for separate modules

Module-III

Group A (40 marks)

Answer any two questions **[2×20=40]**

1. i) State and prove well ordering property of real numbers.

ii) Let S be a subset of $\mathbb{R}$ .Then $\overline{S} = S \cup S'$. S' is the derived set of S.

iii) Let S be a non-empty subset of $\mathbb{R}$, bounded above and T=$\{-x: x \in S\}$. Prove that the set T is bounded blow and inf T= $-\sup S$. (3+4+3)

2. i) When a set is said to be an enumerable set ? Is the set of integers enumerable?

ii) Prove that the set of rational numbers is enumerable.

iii) Let S =$\{(-1)^m + \frac{1}{n} : m \in \mathbb{N}, n \in \mathbb{N}\}$. Find the derived set of S. (1+2+3+4)

3. i) Let G be an open set in $\mathbb{R}$ and S be a subset of $\mathbb{R}$ such that G$\cap S = \emptyset$.Prove that G$\cap S' = \emptyset$.

ii) Prove that a Cauchy sequence of real numbers is convergent.

iii) A sequence $\{u_n\}$ defined by $u_{n+2} = \frac{(u_{n+1}+u_n)}{2}$ for n $\geq$ 1 and 0< $u_1 < u_2$. Prove that the sequence $\{u_n\}$ converges to $\frac{u_1+2u_2}{3}$. (3+4+3)

4. i) A monotone sequence of real numbers having a convergent subsequence with limit l,it is convergent with limit l

ii) Abounded sequence $\{u_n\}$ is convergent iff $\overline{lim} u_n = \underline{lim} u_n$.

iii) If lim$u_n = l$ then show that lim $\frac{u_1+u_2+\cdots\ldots\ldots+u_n}{n} = l$. (4+3+3)

5. i) Let $I = [a, b]$ be a closed and bounded interval and a function $f: I \to \mathbb{R}$ be continuous on I.Then f is bounded on I.

ii) State and prove Intermediate value theorem.

iii) Show that $\lim_{x\to 0} \cos\frac{1}{x}$ (x$\neq$ 0) does not exist. (3+4+3)

6. i) A function $f: [0,1] \to \mathbb{R}$ is continuous on [0,1] and assumes only rational values. If f(1/2)=1/2, prove that f(x)=1/2 for all x $\in$ [0,1].

ii) (a) Prove that the intersection of finite number of open set in Ris an open set and (b) Prove or disprove: the intersection of infinite number of open sets in R is not necessarily an open set.

iii) Let A, B$\subset\mathbb{R}$ and $f: A \to \mathbb{R}$, $g : B \to \mathbb{R}$ be functions such that f(A)$\subset$B. Let $c \in A$ and f is continuous at c and g is continuous at f(c)ϵ B. Then the composite function $gof: A \to \mathbb{R}$ is continuous at c. (3+4+3)

7. i) State and prove sequential criterion for continuity.

ii) Let $f: \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$. Prove that the set $S = \{x \in \mathbb{R}: f(x) > 0\}$ is an open set in $\mathbb{R}$.

iii) Prove or disprove the union of an infinite numbers of closed sets in $\mathbb{R}$ is not necessarily a closed set. (4+3+3)

Group B (10 marks)
Evaluation of Integrals
***Answer any <u>one</u> questions* [1x10=10]**

1. Find the reduction formula for $\int sin^m x cos^n x dx$, m, n being positive integers greater than1. Hence find $\int sin^4 x cos^2 x dx$.

2. Find $\int \frac{dx}{5+4\cos x}$.

3. Find $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x} dx$.

Module-IV
Group A
***Answer any <u>four</u> questions* [4x9=36]**

1. Express the determinant $\begin{vmatrix} b^2+c^2 & ab & ca \\ ab & c^2+a^2 & bc \\ ca & bc & a^2+b^2 \end{vmatrix}$ as a square of a determinant of order 3. Hence determine the value of the determinant.

2. Determine the value of $\begin{vmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{vmatrix}$ by Laplace's expansion in terms of minors of order 2

3. If $A=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that $(A^{-1})^2+A=I$.

4. Define row reduced echelon form of a square matrix. Determine the rank of the matrix $\begin{bmatrix} 1 & -1 & 2 & 0 \\ 2 & 2 & 1 & 5 \\ 1 & 3 & -1 & 0 \\ 1 & 7 & -1 & 1 \end{bmatrix}$ after reducing it into a row-reduced echelon matrix.

5. Define subspace of a vector space V over a field F. Show that the necessary and sufficient condition for a nonempty subset W of a vector space V(F) to be a subspace of V is that $a,b \in F$ and $\underline{\alpha},\underline{\beta} \in W \Rightarrow a\underline{\alpha}+b\underline{\beta}$.

6. Show that the subset $S=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \Re_{2\times 2} : \begin{vmatrix} a & b \\ c & d \end{vmatrix}=0\right\}$ of the set $\Re_{2\times2}$ of all 2×2 real matrices forms a subspace of $\Re_{2\times2}$.

7. Define linear span L(S) of a subset S of a vector space V and specify the conditions for which L(S) will be a basis and of a vector space V. Show whether the set of vectors $S=\{(1,-2,2),\ (3,1,0),\ (-1,2,-4)\}$ forms a basis for $\Re^3$.

8. Determine the values of a and b for which the system of equations

$$x + y + z = 1$$
$$x + 2y - z = b$$
$$5x + 7y + az = b^2$$

admits of (i) unique solution, (ii) infinite number of solutions, (iii) no solution.

9. Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$.

Group B (14 marks)

Answer any <u>One</u> questions **[1×14=14]**

1. Find the directional derivative of $\phi = xy^2z + 4x^2z$ at $(-1,1,2)$ in the direction $2\hat{i} + \hat{j} - 2\hat{k}$.

2. Show that the necessary and sufficient condition for a vector $\vec{\alpha}$ to have a constant direction is $\vec{\alpha} \times \dfrac{d\vec{\alpha}}{dt} = \vec{0}$.

3. (i) If the vectors $\vec{A}$ and $\vec{B}$ are irrotational, then show that the vector $\vec{A} \times \vec{B}$ is solenoidal. (ii) Find $div\ grad\ \phi$, where $\phi = 2x^2y^3z^4$

4. Prove that Curl Curl $\vec{f} = \vec{\nabla}(\vec{\nabla}.\vec{f}) - \nabla^2 \vec{f}$.