Vidyasagar College for Women
Internal Examination 2020
Mathematics (Hons.) MTMA Part-I
Full Marks: 100 Paper-1 Time: 2 Hrs

Group-A

Answer **any two** questions **[2x10]**

1. a) If α, β are the roots of $x^2 - 2x\cos x + 1 = 0$, find the equation whose roots are α^n and β^n. (n is a positive integer)

b) Express $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ as a sum of a symmetric and skew symmetric matrices.

2. a) Prove without expanding that $\begin{vmatrix} a & d & 3a-4d \\ b & e & 3b-4e \\ c & f & 3c-4f \end{vmatrix} = 0$

b) State De Moivre's theorem and apply it to express $\cos 3\theta$ in terms of powers of $\cos\theta$ where θ is real.

3. a) Express Log(1+i) where i= $\sqrt{-1}$ in the form of A+iB where A,B are real numbers.

b) Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

4.a) Find the rank of A= $\begin{pmatrix} 1 & 5 & 9 \\ 4 & 8 & 12 \\ 7 & 11 & 15 \end{pmatrix}$

b) State when a square matrix A is said to be invertible and state a necessary and sufficient condition under which A will be invertible. Show that $(A^{-1})^T = (A^T)^{-1}$

5. Solve by Cardan Method : $x^2 - 12x + 65 = 0$.

6. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$., then show that $A^2 = A^{-1}$.

7. Solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$ whose roots are in G.P.

Group-B

Answer **any one** questions **[1×15]**

8. Reduce the equation $9x^2 + 24xy + 16y^2 - 126x + 82y - 59 = 0$ into canonical form and determine the nature of the conic represented by it.

9. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then prove that pq = -1.

10. PSP' and QSQ' are two perpendicular focal chords of $\frac{l}{r} = 1 - e\cos\theta$, show that

$$\frac{1}{PP'} + \frac{1}{QQ'} = \frac{2-e^2}{2l}.$$

11. When the axes are turned through an angle, the expression $ax+by$ becomes $a_1x + b_1y$ referred to the new axes. Show that $a_1^2 + b_1^2 = a^2 + b^2$ and find the angle of rotation.

12. a) If $ax^2+2hxy+by^2$ transforms into $AX^2+2HXY+BY^2$ under rotation of axes then show that $a+b=A+B$

b) Find the angle of rotation about the origin which will transform the equation $x^2-y^2=4$ into $x'y' = 2$

Group-C

Answer **any two** questions **[2x12]**

13) $y= 2\cos x (\sin x - \cos x)$ show that $(y_{10})_0 = 2^{10}$

14) If $y= \sin(m\sin^{-1}x)$, show that

$(1-x^2)y_2 - xy_1 + m^2y = 0$ and hence prove that

$(1- x^2) y_{n+2} -(2n+1)xy_{n+1} - (n^2- m^2)y_n=0$

15) Find the angle between $x^2 = 4y$ and $y^2 = 4x$.

16. Show that the maximum value of $x^{\frac{1}{x}}$ is $e^{\frac{1}{e}}$.

17. Test the series for convergence or divergence

$$\frac{3}{2} + \frac{4}{2^2} + \frac{5}{2^3} + \cdots + \frac{n+2}{2^n} + \cdots$$

18. A function $f(x)$ is defined in [0, 2] by

$$f(x) = \begin{cases} x^2 + x & \text{for } 0 \le x < 1 \\ 2 & \text{for } x = 1 \\ 2x^3 - x + 1 & \text{for } 1 < x \le 2 \end{cases}$$

Examine the continuity and differentiability of $f(x)$ at $x = 1$.

19. Show that the maximum rectangle inscribable in a circle is a square.

Group-D

Answer **any two** questions **[2x8]**

20. Given two vectors, $\vec{\alpha} = 2\hat{i} + \hat{j} - 3\hat{k}$, $\vec{\beta} = -\hat{j} + 3\hat{k}$, find a unit vector $\vec{\delta}$ which is coplanar with $\vec{\alpha}$ and $\vec{\beta}$ and perpendicular to $\vec{\gamma} = \vec{i} + \vec{j} - \vec{k}$.

21. If $\vec{\alpha} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{\beta} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{\gamma} = 3\hat{i} + \hat{j} - \hat{k}$, find a vector $\vec{\rho}$ which is perpendicular to both $\vec{\alpha}$, $\vec{\beta}$ and satisfies the relation $\vec{\rho}.\vec{\gamma} = 21$.

22. A particle being acted on by constant forces $-2\hat{i} + 5\hat{j} - 2\hat{k}$, $3\hat{i} - 4\hat{j} + \hat{k}$ and $6\hat{i} + \hat{j} - 3\hat{k}$ is displaced from the point (2, 0, 1) to (3, 2, -1). Find the total work done by the forces.

23. Prove by vector method that the perpendicular bisectors of the sides of a triangle are concurrent.

Group-E

Answer **any one** questions **[1x10]**

24) Evaluate $\int \dfrac{2\sin x + 3\cos x}{3\sin x + 4\cos x}$

25) Evaluate $\int \dfrac{dx}{5 + 4\cos x}$

26. Evaluate $\underset{n\to\infty}{Lt}\left[\left(1+\dfrac{1^2}{n^2}\right)\left(1+\dfrac{2^2}{n^2}\right)\left(1+\dfrac{3^2}{n^2}\right)\Lambda\ \Lambda\ \left(1+\dfrac{n^2}{n^2}\right)\right]^{\frac{1}{n}}.$

27. Evaluate: $\int_0^1 \dfrac{\log(1+x)}{1+x^2}\,dx.$

Group-F

28) *Solve* ***any two*** of the following differential equations: [**15**]

a) $(x^2 - yx^2)\,dy + (y^2 + xy^2)\,dx = 0$

b) $(2x + 3y - 6)\,dy = (6x - 2y - 7)\,dx$

c) $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$, given $y = 1$ when $x = 1$.

d) $xdx + y\,dy + \dfrac{xdy - ydx}{x^2 + y^2} = 0$.

e) Solve : $\dfrac{dy}{dx} + \dfrac{y}{x}\log y = \dfrac{y}{x^2}(\log x)^2$